Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name: Theories of Ring and Field

Subject Code:5SC03TRF1 **Branch: M.Sc. (Mathematics)**

Semester: 3 **Date:18/03/2019** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1		**Attempt the Following questions**	**(07)**
	a.	Define: Ring	**1**
	b.	Define: Integral Domain	**1**
	c.	Define: Field	**1**
	d.	Define: Isomorphism of ring	**1**
	e.	Define: Subring	**1**
	f.	Is $Z \times Q$ an integral domain? Justify your answer.	**2**
Q-2		**Attempt all questions**	**(14)**
	a.	State and prove Unique factorization theorem.	**6**
	b.	Prove that every finite integral domain is a field.	**5**
	c.	How many subring of order 7 in $\boldsymbol{Z_{49}}$**?** Justify.	**3**
		OR	
Q-2		**Attempt all questions**	**(14)**
	a.	If a is an element in a commutative ring R with unity then prove that the set $S = \{ra \mid r \in R\}$ is a principal ideal of R generated by a.	**6**
	b.	Prove that characteristic of an integral domain is either zero or prime number.	**5**
	c.	If U is an ideal of R with unity and $1 \in U$ then prove that $U = R$	**3**
Q-3		**Attempt all questions**	**(14)**
	a.	Let F be a field and $f(x), g(x)$ be any two polynomials in $F[x]$ not both of which are zero. Then prove that greatest common divisor $d(x)$ of $f(x)$& $g(x)$ can be expressed in the form $d(x) = m(x)f(x) + n(x)g(x)$ for $m(x), n(x) \in F[x]$.	**6**
	b.	State and prove Remainder theorem.	**4**
	c.	If $f(x) = 3x^7 + 2x + 3$ and $g(x) = 5x^3 + 2x + 6$ be two polynomials over the field $(Z_7, +_7, \times_7)$. Find (i) $\frac{d}{dx}(f(x))$ (ii) $f(x)g(x)$.	**4**
		OR	

Q-3 **Attempt all questions** **(14)**

- **a.** State and prove division algorithm for polynomials over field. **7**
- **b.** Prove that intersection of two left ideal of a ring is again a left ideal of ring. **4**
- **c.** Prove that characteristic of a ring with unity is 0 and $n > 0$ where 0 and n are order of 1 in additive group **3**

SECTION – II

Q-4 **Attempt the Following** **(07)**

- **a.** Define: Left Ideal. **1**
- **b.** Define: Principal Ideal Ring. **1**
- **c.** Define: Euclidean Ring. **1**
- **d.** Define: Fixed field **1**
- **e.** State Factor theorem. **1**
- **f.** Define: Perfect field. **1**
- **g.** Define: Automorphism of field. **1**

Q-5 **Attempt all questions** **(14)**

- **a.** Prove that $F[x]$ is a principal ideal ring. **6**
- **b.** Let R be Euclidean ring. Let a and b be two non-zero elements in R. If b is not unit in R then prove that $d(b) < d(ab)$. **5**
- **c.** Prove that every field is a Euclidean ring. **3**

OR

Q-5 **Attempt all questions** **(14)**

- **a.** Prove that ring of Gaussian integer is a Euclidean ring. **6**
- **b.** Prove that every Euclidean ring is a principal ideal ring. **5**
- **c.** Prove that every Euclidean ring possesses unity. **3**

Q-6 **Attempt all questions** **(14)**

- **a.** State and prove fundamental theorem on Galois theory. **7**
- **b.** State and prove Gauss Lemma. **5**
- **c.** Define: Splitting field. Give one example. **2**

OR

Q-6 **Attempt all Questions**

- **a.** Let K be field of complex number and F be the field of real numbers then prove that K is a normal extension of F. **5**
- **b.** Let K be an extension field of rational numbers Q. Show that any automorphism of K must leave every element of Q. **5**
- **c.** Let G be a subgroup of the group of all automorphisms of a field K. Then prove that the fixed field of G is a subfield of K. **4**